Extremal Problems on the p-Seidel Energy of Graphs

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Abstract

Let G be a simple graph and let $S(G)$ denote its Seidel matrix. If $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of $S(G)$, then, for $p > 0$, the p-Seidel energy of G is defined by

$$\mathcal{E}_p(S(G)) = \sum_{i=1}^{n} |\lambda_i|^p.$$

This notion extends the classical Seidel energy and naturally leads to extremal problems concerning the distribution of Seidel eigenvalues.

In this talk, we discuss several extremal problems for the p-Seidel energy of graphs. For $p > 2$, we determine the minimum p-Seidel energy among graphs of fixed order and characterize the equality case in terms of conference matrices. For $0 < p < 2$, the corresponding inequality is reversed, yielding a maximum result. We then consider regular graphs. In particular, when n is a prime power with $n \equiv 1 \pmod 4$ and the degree is $\frac{n-1}{2}$, we obtain a sharp lower bound related to Paley graphs. We also determine the maximum p-Seidel energy among r-regular graphs of order $n = 2r$, where the extremal graph is the balanced complete bipartite graph $K_{r,r}$. The proofs combine spectral properties of Seidel matrices with power-mean inequalities and constrained optimization.