Isometries of Wasserstein spaces

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Due to its nice theoretical properties and an astonishing number of applications via optimal transport problems, probably the most intensively studied metric nowadays is the p-Wasserstein metric. Given a complete and separable metric space X and a real number $p \geq 1$, one defines the p-Wasserstein space $\mathcal{W}_p(X)$ as the collection of Borel probability measures with finite p-th moment, endowed with a distance which is calculated by means of transport plans.

The main aim of our research project is to reveal the structure of the isometry group $\mathrm{Isom}(\mathcal{W}_p(X))$. Although $\mathrm{Isom}(X)$ embeds naturally into $\mathrm{Isom}(\mathcal{W}_p(X))$ by push-forward, and this embedding turned out to be surjective in many cases (see e.g. [1]), these two groups are not isomorphic in general. Kloeckner computed in [2] the isometry group of the quadratic Wasserstein space over the real line. It turned out that this group is extremely rich: it contains a flow of wild behaving isometries that distort the shape of measures. Following this line of investigation, we computed $\mathrm{Isom}(\mathcal{W}_p(\mathbb{R}))$ and $\mathrm{Isom}(\mathcal{W}_p([0,1]))$ for all $p \geq 1$. In this mini-series of talks we will survey first some of the earlier results in the subject, and then we will present the key results of our recent manuscript [3].

References

[1] J. Bertrand and B. Kloeckner, *A geometric study of Wasserstein spaces: isometric rigidity in negative curvature*, International Mathematics Research Notices, 2016 (5), 1368–1386.

[2] B. Kloeckner, *A geometric study of Wasserstein spaces: Euclidean spaces*, Annali della Scuola Normale Superiore di Pisa - Classe di Scienze, Serie 5, Tome 9 (2010) no. 2, 297–323.

[3] Gy. P. Gehér, T. Titkos, D. Virosztek, *Isometric study of Wasserstein spaces – the real line*, Manuscript under revision.