`Question:` I had a question concerning a basic step in Iskovskikh's Fano classification. The question has to do with the list of Fanos V where $-K_V$ is not spanned.

After resolving the base locus, we have a map to a minimal variety. For simplicity, assume it's a ruled surface W. The idea is then to study the pull back Z of the exceptional section of W. Key point in the proof is to show that $h^1(\mathcal{O}_Z)$ of this surface is less or equal to one, and this is the point I do not understand. The reference in your more than useful book is Reid's Lemma, by which he filled the gap in Shokurov's general elephant proof, but I do not see why this Lemma applies. What one needs is that Z has at most double curves as singularities.

`Answer:` You are absolutely right, in the book the proof is incomplete. The point is that the surface C' can have cuspidal singularities along a curve which has to be a section of $C' \to \varphi(C')$. The original proof is rather long. Here is an easy argument.

Since $C = C' \cap E$,

$$N_{C/\tilde{X}} = N_{C/C'} \oplus N_{C/E}.$$

Hence $N_{C'/\tilde{X}}|_C \simeq N_{C/E}$ is negative. In particular, C' is not nef. Note that $-K_{\tilde{X}}$ is nef and the Mori cone is polyhedral and generated by contractible extremal rays. So there is an extremal ray R such that $C' \cdot R < 0$. Since C' is nef over W, R is K-negative. By the classification of extremal rays C' is normal.